ONE BY ONE EMBEDDING THE CROSSED HYPERCUBE INTO PANCAKE GRAPH

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ABSTRACT

Let *G* and *H* be two simple undirected graphs. An *embedding* of the graph *G* into the graph H is an injective mapping *f* from vertices of *G* to the vertices of *H*. The *dilation* of embedding is the maximum distance between *f*(*u*), *f*(*v*) taken over edges (*u*, *v*) of *G*. The Pancake graph is one as viable interconnection scheme for parallel computers, which has been examined by a number of researchers. The Pancake was proposed as alternatives to the hypercube for interconnecting processors in parallel computer. Some good attractive properties of this interconnection network include: vertex symmetry, small degree, a sub-logarithmic diameter, extendability, and high connectivity (robustness), easy routing and regularity of topology, fault tolerance, extensibility and embeddability of others topologies. In this paper, we give a construction of one by one embedding of dilation 5 of crossed hypercube into Pancake graph.

Keywords: Embedding, n-dimensional Crossed Hypercube, n-dimensional Pancake, dilation.

1 INTRODUCTION

In the field of interconnection networks, the study of graph embedding is motivated by the problem: Efficient simulation of interconnection networks and parallel algorithms on a different network, layout of circuits in VLSI. Akers and Krishnamurthy 1990 [1] proposed the Pancake as an attractive alternative to the hypercube and their variations for interconnecting processors in large scale parallel computers. This graph belongs to the family of Cayley graphs. It has very many interesting properties: small diameter and fixed degree, (*n*-1) regular, high connectivity, vertex symmetry, Hamiltonian, fault tolerance, pancyclicity, extensibility and embeddability of others topologies. Akers and al. 1990[1], Kanevsky and Feng 1995 [16], Hwang and Chen 2000 [15], Hung and Al. 2003 [10], Heydari and Sudborough 1997 [8], Hsieh and Chang 2006 [12], Hsieh and Lee 2009 [13], 2010 [14]. The embedding capabilities are important in evaluating an interconnection network. Let *G* and *H* denote the simple undirected graphs. In general, an *embedding* of the graph *G* into the graph *H* is an injective mapping *f* from vertices of *G* to the vertices of *H* together with mapping $Path_j$ which assigns to each edge (*u*, *v*) of *G* to a path between *f*(*u*) and *f*(*v*) in *H*. Let *dil*(*f*) denote the *dilation* of given embedding *f*., is defined to be the maximum of length { $Path_j(u,v)$: (u,v) € *E*(*G*)}. Menn and Somani 1992 [20], Fan 2002 [5], Qiu 1992 [24], Fang and Hsu 2000 [6], Hsieh and al 1999 [11], Lin and al. 2008 [18], 2010 [19], Aschheim and al. 2012 [2], Femmam and al. 2012 [20]. To compare with crossed hypercube, the pancake offers a good and simple simulation of the others interconnection networks Sennoussi and Lavault 1997 [22].

In this paper, we consider the one by one embedding of *n*-dimensional Crossed hypercube into the *n*-dimensional Pancake graph. Our goal is to construct the dilation 5 one by one embedding *n*-dimensional Crossed hypercube into *n*-dimensional Pancake graph. The paper is organized as follows: We introduce some definitions and properties of Crossed hypercube and Pancake graph in the preliminaries. In the section 3, we present the construction of one by one embedding of *n*-dimensional Crossed hypercube into *n*-dimensional Pancake graph. In the section 4 we show that the dilation of one by one embedding Crossed hypercube into Pancake is equal 5. Finally, we give our conclusion in the section 5.

2 PRELIMINARIES

Definition 1. Let *n* be a positive integer. The star graph S_n and Pancake graph $P_n = (G_n, E_n)$ of dimension *n* are graphs whose vertex set G_n consists of all of permutations $G_{n=} ((g_1,g_2,\ldots,g_n) \mid g_i \in I = \{1,2,\ldots n\}., g_i \neq g_i$ for $i \neq j)$. The ith position of the vertex $x_1x_2\ldots x_n$ of star or Pancake will be referred to as the i^{th} coordinate of the vertex. In the star graph S_n a vertex $x_1x_2\ldots x_n$ is adjacent to the vertices obtained $x_ix_2\ldots x_{i-1}x_1x_{i+1}\ldots x_n$ for $2 \leq i \leq n$. In the Pancake graph P_n a vertex $x_1x_2\ldots x_n$ is adjacent to the vertices $x_ix_{i-1}\ldots\ldots x_1x_{i+1}\ldots x_n$ for $2 \leq i \leq n$. i.e. vertices obtained by reversing the order of the symbols in the first *i* coordinates of the vertex for $2 \leq i \leq n$. $E_n = \{((g_ig_{i-1}\ldots g_1g_{i+1}\ldots g_n),(g_ig_2\ldots g_{i-1}g_1g_{i+1}\ldots g_n).\ g_i \in G_n$ for $2 \leq i \leq n.)\}$, $|E_n|$=.(n-1)n!/2.. Thus, the star or Pancake graph of dimension *n* has *n*! vertices and each of its vertices is adjacent to *n*-1 other vertices. The graph P_n is made of *n* copies of G_{n-1} namely $P_n[n, k]$ for $1\leq k \leq n$. Considering each $P_n[n, k]$ as a super node. It follows that $P_n[n,s]$, $P_n[n,t]$ are connected by a collection of edges of the form

$((t,g_2,g_3,.....,g_{n-1},s),\ (s,g_{n-1},.....,g_2,t))$ Thus, the are $(n$-2)! edges connecting $P_n[n,s]$ and $P_n[n,t]$ Kanevsky and Feng, 1995 [15] The n-pancake P_n is a complete graph on the super nodes connected by the super edges.

Definition 2. The n-dimensional hypercube Q_n and the crossed hypercube CQ_n have a same set of vertices. We represent the address of each vertex in Q_n (CQ_n) as a binary string of length n. In such away, we don't distinguish between vertices and their binary address. In Q_n two vertices are adjacent if and only if their binary labels differ only in one bit position. For CQ_n, adjacency requirement are little more involved.

Two binary strings $x = x_1x_0$ and $y = y_1y_0$ of length two are pair-related if and only if (x,y) € {(00,00),(10,10),(01,11),(11,01)}.

The n-dimensional Crossed hypercube CQ_n is recursively defined as follows: CQ_n is the complete graph base on two vertices labeled 0 and 1.K. Efe 1992 [4]. CQ_n consists of two subcubes $0CQ_{n-1}$ and $1CQ_{n-1}$. The most significant bit of the labels of the vertices in $0CQ_{n-1}$ and $1CQ_{n-1}$ is 0(1). U is the set of vertices $u= u_{n-1}u_{n-2}\ldots u_1u_0$ € $0CQ_{n-1}$ with $u_{n-1} = 0$ and $v= v_{n-1}v_{n-2}\ldots v_1v_0$ € $1CQ_{n-1}$ with $v_{n-1} = 1$ are joined by an edge in CQ_n if and only if:

$u_{n-2} = v_{n-2}$ if n is even

$(u_{2i+1}\ u_{2i},\ v_{2i+1}\ u_{2i})$ are pair-related.

The n-dimensional Crossed hypercube CQ_n as an alternative of the hypercube, has the same number of vertices and degree as the n-dimensional hypercube. The Crossed hypercube is a variation of hypercube which is derived with some twisted edges. Due to these twisted edges, the diameter of CQ_n is only half of the hypercube one. Nice properties include relatively small degree, embedding capabilities, scalability, robustness and the fault tolerant of hamiltonicity of CQ_n. Huang et al., 2002 [23], Hsieh et al., 1999 [16]). The multiply twisted hypercube graph is not vertex-transitive for $n \geq 5$ (Kulasinghe and Bettayeb, 1995 [17]).

3 EMBEDDING ONE BY ONE *N*-DIMENSIONAL CROSSED HYPERCUBE INTO *N*-DIMENSIONAL PANCAKE GRAPH.

In this section, we present a new function, the one by one embedding n–dimensional Crossed hypercube denoted CQ_n into n-dimensional Pancake graph denoted by P_n. The main steps of one by one embedding are as follows:

- Find the first node of CQ_n and the first node of P_n. *Example* 00000 of CQ_4 and 1234 of P_4.
- One by one embedding vertex of CQ_n into P_n
- One by one embedding all edges of CQ_n into paths P_n.

3.1 One by one embedding vertex of CQ_n into P_n.

The One by one embedding vertex of CQ_n into P_n.is done in the following way:

The basic function of this one by one embedding vertex is produced as follows: CQ_4 is made recursively by two copies of CQ_3. One copy is prefixed bi 0 ($0CQ_3$) and the other one is prefixed by 1 ($1\ CQ_3$). The P_4 is made by four copies of P_3 named $P_4[4,k]$ for k = 1.4. The one by one applies all following actions:

a) The vertex of $0CQ_3$ are respectively embedded into $P_4[4,4]$ and $P_4[4,1]$ using:
X=PremG(node), inv1(X), flip(X), flip(inv1(X)).
Y=Inv4(X),), flip(X), inv1(X), inv1(flip(X).

b) The vertex of $1CQ_3$ are respectively embedded into $P_4[4,2]$ and $P_4[4,3]$ using:
Z=Inv4(Inv1(Inv1(z))), inv1(Y), flip(Y), flip(inv1(Y).
T= Inv4(Y), flip(T), inv1(T), inv1(flip(T)).

Remarque that only the first action is changed.

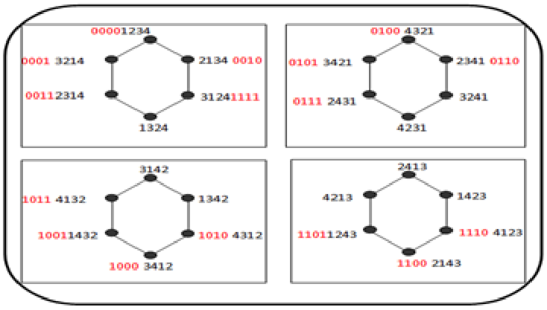

Figure 1: The one by one embedding vertex of CQ_4 into P_4

The case for $n \geq 5$. The n-dimensional crossed hypercube CQ_n is produced the composition of two copies of CQ_{n-1} . The first one is prefixed by 0 ($0CQ_{n-1}$) and the second is prefixed by 1($1CQ_{n-1}$). The n-Pancake P_n is made by n-1 copies of P_{n-1}.The are two stated situations. The first one is when n is odd, we use two components. Example shown in figure 2.

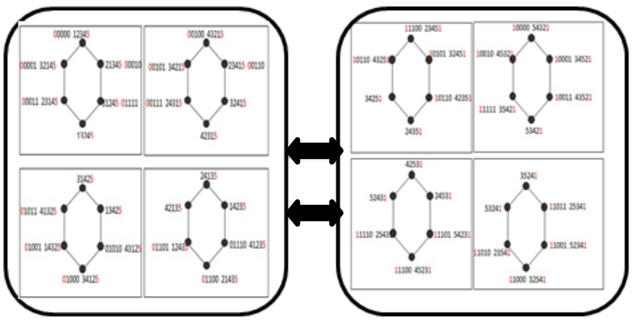

Figure 2: One by one embedding $0CQ_4$ into $P_5[5,1]$

The first for one by one embedding all nodes of $0CQ_{n\text{-}1}$ uses the actions of the basic function of the one by one embedding with X=PremG(node) (the first node) in the super node $P_{n\text{-}1}[n\text{-}1, n\text{-}1]$.and in the second one for one by one embedding vertex of $1CQ_{n\text{-}1}$, we applied the actions of the basic function of the one by one embedding with Y=Inv(X) (the first node) in $P_{n\text{-}1}[n\text{-}1,1]$.

The second situation is when n is even, we use four super nodes of P_n. Example shown in figure 3. We embed in the first super node of $P_{n\text{-}1}$ all nodes of $00CQ_{n\text{-}2}$ are embedded into $P_{n\text{-}1}[n\text{-}1, n\text{-}1]$ using the actions of basic function one by one embedding CQ_n into $P_{n\text{-}}1$ with the first node X=PremG(node). In the second super node $P_{n\text{-}1}[n\text{-}1,1]$ of P_n, all nodes of $01CQ_{n\text{-}2}$ are one by one embedded with the same basic actions for Y= Inv(X). all nodes $10CQ_{n\text{-}2}$ are embedded into the third component $P_{n\text{-}1}[n\text{-}1, 2]$ using the same actions with Z= Inv1(Inv((Inv1(PremG(nœud))), finally, we embed all vertex of $11CQ_{n\text{-}2}$ in the fourth super node $P_{n\text{-}1}[n\text{-}1, 3]$ with T= Inv((flip(inv4(PremG(nœud))) and using the actions of the basic function, Figure 3.

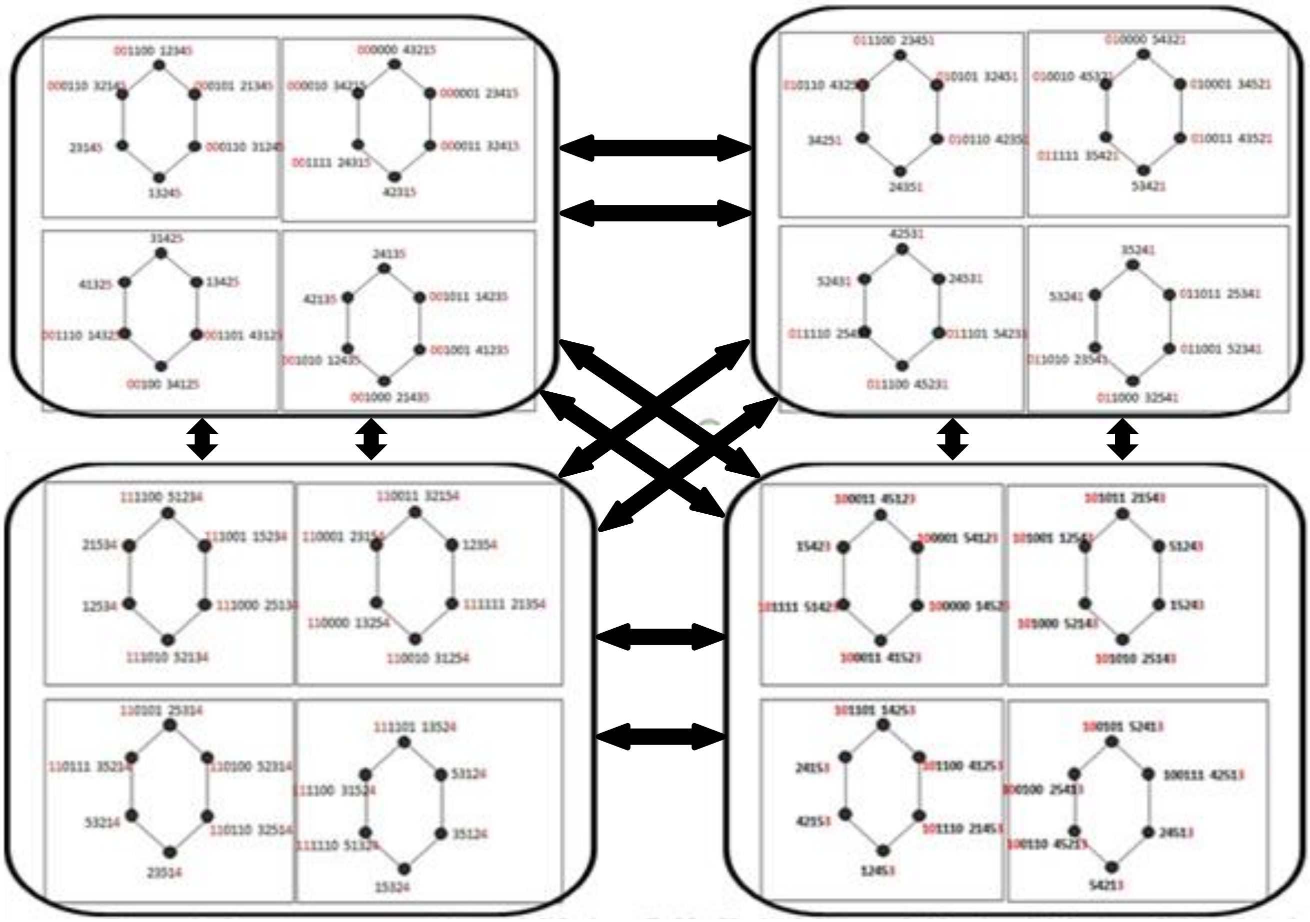

Figure 3 : One by one embedding 00CQ$_4$ into P$_5$[5,5], 01CQ4 into P$_5$[5,1], 10CQ$_4$ into P$_5$[5,3] and 11CQ$_4$ into P$_5$[5,4]

3.2 One by one embedding edges of CQ_n into P_n.

There are two stated situations**.** The first one is when the paths are in the same P_4 of any super node of P_n. the second is when the path is between to P_4 of any component of P_n.

In the first situation, the one by one embedd*in*g edges of CQ_n into paths of P_n uses two ways realizing as follows:

The first one is to one by one embedding all edges of CQ_n with extremities are *Pref*aaaa-*prefbbbb* that *a,b* Є [0,1] in the other word the embedded path is the same P_3 or between the different P_3 of any super node of P_n. We use the actions depicted in Table 1.

The second one is the one by one embedding edges between two P_4 of any component of P_n. In this situation, we use actions depicted in, Table 3, Table 4 and Table 5.

Table 1: One by one embedding edges of CQ_n into paths between the P_3 in the same P_4 of any super node of P_n

Edges in CQ_n	Paths in P_m	Dilation
Pre00f00- Pref0001	x1x2x3x4*Suf*$_1$-x2x1x3x4*Suf$_1$*	1
Pref 0000-Pref0010	x1x2x3x4*Suf*1-x2x1x3x4*Suf1*	1
Pref 0001-*pref*0011	x3x2x1x4*Suf1*-x2x3x1x4*Suf1*	1
Pref 0010-*pref*0011	x2x1x3x4*Suf*1-x1x2x3x4*Suf*1-x3x2x1x4*Suf*1-x2x3x1x4*Suf1*	3
Pref 0100-*pref*0101	x4x3x2x1*Suf*1-x3x4x2x1*Suf1*	1
Pref 0100-*pref*0110	x4x3x2x1*Suf*1-x2x3x4x1*Suf1*	1
Pref 0101-*pref*0111	x3x4x2x1*Suf1* -x2x4x3x1*Suf1*	1
Pref 0110-*pref*0111	x2x3x4x1*Suf*1-x3x2x4x1*Suf*1-x4x2x3x1*Suf1*-x2x4x3x1*Suf1*	3
Pref 1000-*pref*1001	x3x4x1x2*Suf*1-x1x4x3x2*Suf1*	1
Pref 1000-*pref*1010	x3x4x1x2*Suf*1-4x3x1x2*Suf1*	1
Pref 1001-*pref*1011	x1x4x3x2 *Suf*1-x4x1x3x2 *Suf1*	1
Pref 1010-*pref*1011	x4x3x1x2 *Suf1* -x3x4x1x2 *Suf*1-x1x4x3x2 *Suf*1-x4x1x3x2*Suf1*	3
Pref 1100-*pref*1101	x2x1x4x3*Suf*1-x1x2x4x3*Suf1*	1
Pref 1100-*pref*1110	x2x1x4x3*Suf*1-x4x1x2x3*Suf1*	1
Pref 1101-*pref*1111	x1x2x4x3*Suf*1-x4x2x1x3*Suf*1-x3x1x2x4*Suf1*	1
Pref 1110-*pref*1111	x4x1x2x3*Suf1* -x1x4x2x3*Suf*1-x2x4x1x3*Suf1*-x4x2x1x3*Suf*1-x3x1x2x4*Suf1*	4

Table 2: One by one embedding edges between CQ_n^{00} and CQ_n^{10} of CQn into Paths between two components of P_3 suffixed by Suf1 and Suf2

Edges between CQ_n^{00} and CQ_n^{10}	Paths between two components of P_4	Dilation
*Pref*0000- *Pref*1000	x1x2x3x4 *Suf1*-x2x1x3x4 *Suf*1-inv(suf1)x4x3x2x1 *Suf*2-flip(inv(suf1))x3x4x1x2 *Suf2*	3
Pref 0001-*pref*1011	x3x2x1x4*Suf*1-x2x3x1x4*Suf*1-inv(suf1)x4x1x3x2*Suf2*	2
Pref 0010-*pref*1010	x2x1x3x4*Suf*1-inv(suf1)x4x3x1x2*Suf2*	1
Pref 0011-*pref*1001	x2x3x1x4*Suf*1-inv(suf1)x4x1x3x2*Suf*2-flip inv(suf1)x1x4x3x2*Suf2*	3

Table 3: One by one embedding edges between CQ_n^{00} and CQ_n^{01} of CQ_n into Paths between two components of P_3 *suffixed by* Suf1 *and* Suf2.

Edges between CQ_n^{00} and CQ_n^{01}	Paths between two components of P_4	Dilation
Pref 0000-*pref*0100	x1x2x3x4 *Suf*1-inv(suf1)x4x3x2x1*Suf2*	1
Pref 0001-*pref*0111	x3x2x1x4 *Suf*1-x2x3x1x4 *Suf*1-x1x3x2x4*Suf*1-inv(suf1)x4x2x3x1 *Suf2* -flip inv(suf1)x2x4x3x1*Suf2*	4
Pref 0010-*pref*0110	x2x1x3x4 *Suf*1-x1x2x3x4 *Suf*1–inv(*Suf*$_1$)x4x3x2x1 *Suf*2-flip(inv(suf1))x3x4x2x1*Suf2*	3
Pref 0011-*pref*0101	x2x3x1x4*Suf*1-x3x2x1x4*Suf1*-x1x2x3x4*Suf*1-inv(suf1)x4x3x2x1*Suf*2-flip inv(suf1)x3x4x2x1*Suf2*	4

Table 4: One by one embedding edges between CQ_n^{10} and CQ_n^{11} of CQ_n into Paths between two components of P_3 *suffixed by* Suf1 *and* Suf2.

Edges between CQ_n^{10} and CQ_n^{11}	Paths between two components of P_4	Dilation
Pref 1000-*pref*1100	x3x4x1x2 *Suf1* -x2x1x4x3 *Suf2*	1
Pref 1001-*pref*1111	x1x4x3x2 *Suf* 1-x4x1x3x2 *Suf* 1-x2x3x1x4 *Suf* 2-x1x3x2x4 *Suf* 2-x3x1x2x4 *Suf2*	4
Pref 1010-*pref*1110	x4x3x1x2 *Suf*1-x3x4x1x2 *Suf* 1-x2x1x4x3 *Suf* 2-x4x1x2x3 *Suf2*	3
Pref 1011-*pref*1101	x4x1x3x2 *Suf* 1-x1x4x3x2 *Suf* 1-x3x4x1x2 *Suf1* -x2x1x4x3 *Suf* 2-x1x2x4x3 *Suf2*	3

Table 5: One by one embedding edges between CQ_n^{01} and CQ_n^{11} of CQ_n into Paths between two components of P_3 *suffixed by* Suf1 *and* Suf2.

Edges between CQ_n^{01} and CQ_n^{11}	Paths between two components of P_4	Dilation
Pref 0100-*pref*1100	x4x3x2x1*Suf*1-x3x4x2x1 *Suf1*–inv(suf1)x1x2x4x3*Suf2*-flip inv(suf1)x2x1x4x3*Suf2*	3
Pref 0101-*pref*1111	x3x4x2x1*Suf1*–inv(suf1)x1x2x4x3*Suf2*-x4x2x1x3*Suf*2-x3x1x2x4*Suf2*	3
Pref 0110-*pref*1110	x2x3x4x1 *Suf1*-x3x2x4x1*Suf*1-inv(suf1)x1x4x2x3*Suf2*-flip inv(suf1)x4x1x2x3*Suf2*	3
Pref 0111-*pref*1101	x2x4x3x1*Suf1*-x3x4x2x1*Suf*1-inv(suf1)x1x2x4x3*Suf2*	2

1. In the second situation, the one by one embedding edges of CQ_n into paths of P_n uses two cases realizing as follows:

a) The first way is to one by one embedding all edges of CQ_n with extremities are *Prefaaaaa-Prefbbbbb* that a,b ∈ [0,1] in other word the embedded path is between two different component P_4 of the same P_5 of any super node of P_n, when n is odd for any $n \geq 5$. We use in this case the following actions depicted in Table 6, Table 7.

b) The second way is to one by one embedding all edges of CQn with extremities are Prefaaaaaa-Prefbbbbbb that a,b ∈ [0,1] in other word the embedded path is between four different component P4 of the same P5 of any super node of Pn , when n is even for any n ≥ 5. We use in this case the following actions depicted in Table 8, Table 9, Table 10, Table 11.

Table 6: One by one embedding edges between CQ_n^{00} ***and*** CQ_n^{10} ***of*** **CQ_n** ***into Paths between two components of*** **P_4** ***suffixed by*** **Suf1** ***and*** **Suf2.**

Edges between CQ_5^{00} and CQ_5^{10}	Paths between two components of P_4	Dilation
0*Pref* 0000- 1*Pref* 0000	x1x2x3x4x5 *Suf* 1-x5x4x3x2x1 *Suf2*	1
*0Pref*0001- 1*Pref*0011	x3x2x1x4x5 *Suf1*-x1x2x3x4x5 *Suf* 1-x5x4x3x2x1 *Suf* 2-x3x4x5x2x1 *Suf* 2-x4x3x5x2x1 *Suf2*	4
0Pref 0010- 1*Pref*0010	x2x1x3x4x5 *Suf* 1-x1x2x3x4x5 *Suf* 1-x5x4x3x2x1 *Suf* 2-x4x5x3x2x1 *Suf2*	3
0Pref 0011-1 *Pref*0001	x2x3x1x4x5 *Suf* 1-x3x2x1x4x5 *Suf* 1-x1x2x3x4x5 *Suf* 1-x5x4x3x2x1 *Suf* 2-x3x4x5x2x1 *Suf2*	4
*0Pref*0100- 1*Pref*0100	x4x3x2x1x5 *Suf*1-x1x2x3x4x5 *Suf* 1-x5x4x3x2x1 *Suf* 2-x2x3x4x5x1 *Suf2*	3
*0Pref*0101- *1Pref*10111	x3x4x2x1x5 *Suf* 1-x1x2x4x3x5 *Suf* 1-x5x3x4x2 *Suf*1-x2x4x3x5x1 *Suf* 2-x4x2x3x5x1 *Suf2*	4
*0Pref*0110- 1*Pref*0110	x2x3x4x1x5 *Suf* 1-x1x4x3x2x5 *Suf* 1-x5x2x3x4x1 *Suf* 2-x4x3x2x5x1 *Suf2*	3
*0Pref*0111- 1*Pref*10101	x2x4x3x1x5 *Suf* 1-x4x2x3x1x5 *Suf* 1-x1x3x2x4x5 *Suf* 1-x5x4x2x3x1 *Suf2*-x3x2x4x5x1 *Suf2*	4

Table 7: The one by one embedding edges between CQ_{n-1}^{0} ***and*** CQ_{n-1}^{1} ***of*** **CQ_n** ***into Paths between two components of*** **P_4** ***suffixed by*** **Suf1** ***and*** **Suf2.**

Edges between CQ_5^{01} and $C1$	Paths between two components of P_4	Dilation
0Pref 1000- 1Pref1000	3x4x1x2x5 *Suf* 1-x1x4x3x2x5 *Suf1*-x5x2x3x4x1 *Suf* 2-x3x2x5x4 *x1Suf2*	3
0Pref 1001- 1Pref1011	x1x4x3x2x5 *Suf1*-x5x2x3x4x1 *Suf* 2-x2x5x3x4*x1Suf2*	2
0Pref 1011- *1Pref* 1001	x4x1x3x2x5 *Suf* 1-x1x4x3x2x5 *Suf* 1-x5x2x3x4x1 *Suf2*	2
*0Pref*1010- *1Pref* 1010	x4x3x1x2x5 *Suf* 1-x3x4x1x2x5 *Suf* 1-x1x4x3x2x5 *Suf* 1-x5x2x3x4x1 *Suf2*-x3x2x5x4x1 *Suf* 2-x2x3x5x4x1 *Suf2*	5
0Pref 1100- 1Pref1100	x2x1x4x3x5 *Suf* 1-x3x4x1x2x5 *Suf* 1-x1x4x3x2x5 *Suf* 1-x5x2x3x4x1 *Suf* 2-x3x2x5x4x1 *Suf* 2-x4x5x2x3x1 *Suf2*	5
0Pref1101- 1Pref1111	x1x2x4x3x5 *Suf* 1-x5x3x4x2x1 *Suf* 2-x3x5x4x2x1 *Suf2*	2
*0Pref*1110- 1 *Pref*1110	x4x1x2x3x5 *Suf* 1-x3x2x1x4x5 *Suf* 1-x1x2x3x4x5 *Suf1*-x5x4x3x2x1 *Suf* 2-x3x4x5x2x1 *Suf2*-x2x5x4x3x1 *Suf2*	5
0Pref1111- 1Pref11101	x3x1x2x4x5 *Suf* 1-x1x3x2x4x5 *Suf* 1-x5x4x2x3x1 *Suf2*	2

Table 8: One by one embedding edges between CQ_{n-2}^{00} ***and*** CQ_{n-2}^{10} ***of*** **CQ_n** ***into Paths between two components of*** **P_4** ***suffixed by*** **Suf1** ***and*** **Suf3.**

Edges between CQ_{n-2}^{00} and CQ_{n-2}^{10}	Paths between two components of P_4	Dilation
00Pref 0000-10*Pref*0000	x1x2x3x4x5 Suf_1-x3x2x1x4x5 Suf_1-x5x4x1x2x3 *Suf3*-x1x4x5x2x3 *Suf3*	3
00Pref 000001-10*Pref*0011	x3x2x1x4x5 *Suf*1-*x*5x4x1x2x3 *Suf*3-x4x5x1x2x3 *Suf3*	2
00Pref 000010-10*Pref*0010	x2x1x3x4x5 *Suf1*-x1x2x3x4x5 *Suf* 1-x3x2x1x4x5 *Suf1*-x5x4x1x2x3 *Suf* 3-x1x4x5x2x3 *Suf*3-x4x1x5x2x3 *Suf3*	5
00Pref 000011-10*Pref* 0001	x2x3x1x4x5 *Suf1*-x3x2x1x4x5 *Suf* 1-x5x4x1x2x3 *Suf3*	2
00Pref 000100- 10Pref0100	x4x3x2x1x5 *Suf* 1-x1x2x3x4x5 *Suf* 1-x3x2x1x4x5 *Suf* 1-x5x4x1x2x3 *Suf3*-x1x4x5x2x3*Suf3*-x2x5x4x1x3 *Suf3*	5
00Pref 000101- 10*Pref* 0111	x3x4x2x1x5 *Suf* 1-x5x1x2x4x3 *Suf3*-x1x5x2x4x3*Suf3*-x4x2x5x1x3*Suf3*	3
00Pref 000110- 10*Pref*0110	x2x3x4x1x5 *Suf*1-x1x4x3x2x5 *Suf1*-x3x4x1x2x5 *Suf*1-x5x2x1x4x3*Suf3*-x1x2x5x4x3 *Suf*3-x4x5x2x1x3 *Suf3*	5

00Pref 000111- 10Pref 0101	x2x4x3x1x5 Suf 1-x1x3x4x2x5 Suf 1-x3x1x4x2x5 Suf 1-x5x2x4x1x3 Suf3	3
00Pref 001000- 10Pref1000	x3x4x1x2x5 Suf 1-x5x2x1x4x3 Suf3	1
00Pref 001001- 10Pref 1011	x1x4x3x2x5 Suf1-x3x4x1x2x5 Suf1-x5x2x1x4x3 Suf3-x1x2x5x4x3 Suf3-x2x1x5x4x3 Suf3	4
00Pref 001010- 10Pref1010	x4x3x1x2x5 Suf 1-x3x4x1x2x5 Suf1-x5x2x1x4x3 Suf3-x2x5x1x4x3Suf3	3
00Pref 001011-10Pref1001	x4x1x3x2x5Suf 1-x1x4x3x2x5Suf1-x3x4x1x2x5Suf1-x5x2x1x4x3 Suf3-x1x2x5x4x3Suf3	4
00Pref 001100-10Pref1100	x2x1x4x3x5Suf 1-x3x4x1x2x5Suf 1-x5x2x1x4x3Suf3-x4x1x2x5x3Suf3	3
00Pref 001101-10Pref1111	x1x2x4x3x5Suf1-2x1x4x3x5Suf1-x4x1x2x3x5Suf 1-x1x4x2x3x5Suf1-x3x2x4x1x5 Suf1-x5x1x4x2x3Suf3	5
00Pref 001110-10Pre01110	x4x1x2x3x5Suf1-x3x2x1x4x5Suf 1-x5x4x1x2x3Suf3-x2x1x4x5x3Suf3	3
00Pref 1111-10Pre01101	x3x1x2x4x5Suf1-x4x2x1x3x5Suf 1-x2x4x1x3x5Suf1-x3x1x4x2x5Suf1-x5x2x4x1x3Suf3-x1x4x2x5x3Suf3	5

Table 9: One by one embedding edges between CQ^{10}_{n-2} and CQ^{11}_{n} of CQ$_{n-2}$ into Paths between two components of P$_4$ suffixed by Suf3 and Suf4.

Edges between CQ^{10}_{n-2} and CQ^{11}_{n-2}	Paths between two components of P_4	Dilation
10Pref100000- 11Pref0000	x1x4x5x2x3 Suf3-x3x2x5x4x1 Suf4-x4x5x2x3x1 Suf4-x1x3x2x5x4 Suf4	3
10Pref100001- 11Pref0011	x5x4x1x2x3 Suf3-x4x5x1x2x3 Suf3-x3x2x1x5x4 Suf4	2
10Pref100010- 11Pref0010	x4x1x5x2x3 Suf3-x1x4x5x2x3 Suf3-x2x5x4x1x3 Suf3-x4x5x2x1x3 Suf3 -x3x1x2x5x4 Suf4	4
10Pref100011- 11Pref0001	x4x5x1x2x3 Suf3-x3x2x1x5x4 Suf4-x2x3x1x5x4 Suf4	2
10Pref100100- 11Pref0100	x2x54x1x3 Suf3-x5x2x4x1x3 Suf3-x3x1x4x2x5 Suf4-x4x1x3x2x5 Suf4 -x5x2x3x1x4 Suf4	4
10Pref100101- 11Pre10111	x5x2x4x1x3 Suf3-x1x4x2x5x3 Suf3 -x4x1x2x5x3 Suf3 -x3x5x2x1x4 Suf4	3
10Pref100110- 11Pref0110	x4x5x2x1x3 Suf3 -x2x5x4x1x3 Suf3-x1x4x5x2x3 Suf3-x4x1x5x2x3 Suf3 -x32x5x1x4 Suf4	4
10Pref100111- 11Pref0101	x4x2x5x1x3 Suf3-x3x1x5x2x4 Suf 4-x1x3x5x2 x4Suf4–x2x5x3x1x4 Suf4	3
10Pref101000- 11Pref 1000	x5x2x1x43 Suf3 -x2x5x1x4x3 Suf3-x3x4x1x5x2 Suf4 -x4x3x1x5x2 Suf4 -x2x5x1x3x4 Suf4	4
10Pref101001- 11Pref1011	x1x2x5x4x3 Suf 3-x2x1x5x4x3 Suf3 -x4x5x1x2x3 Suf3-x3x2x1x5 x4 Suf4 -x5x1x2x3x4 Suf4	4
10Pref101011- 11Pref1001	x2x1x5x4x3 Suf3-x4x5x1x2x3 Suf3 -x3x2x1x5x4 Suf4-x5x1x2x3x4 Suf4-x1x5x2x3x4 Suf4	4
10Pref101010- 11Pref1010	x2x5x1x4x3 Suf3 -x5x2x1x4x3 Suf3 -x3x4x1x2x5 Suf3-x4x3x1x2 x5 Suf4-x5x2x1x3x4 3Suf4	4
10Pref101100- 11Pref1100	x4x1x2x5x3 Suf3-x1x4x2x5x3 Suf3-x5x2x4x1x3 Suf3-x4x2x5x1x3 Suf3-x3x1x5x2x4 Suf3	4
10Pref101101- 11Pref1111	x1x4x2x5x3 Suf3 -x4x1x2x5x3 Suf3-x2x1x4x5x3 Suf3-x3x5x4x1x2 Suf4-x4x5x3x1x2 Suf4-x2x1x3x5x4 Suf4	5
10Pref101110- 11Pref1110	x2x1x4x5x3 Suf3 -x1x2x4x5x3 Suf3-x4x2x1x5x3 Suf3-x3x5x1x2x4 Suf4-x1x5x3x2x4 Suf4-x5x1x3x2x4 Suf4	5
10Pref101111- 11Pref1101	x5x1x4x2x3 Suf3-x1x5x4x2x3 Suf3-x2x4x5x1x3 Suf3 -x4x2x5x1x3 Suf3-x3x1x5x2x4 Suf4-x1x3x5x2x4 Suf4	5

Table 10: One by one embedding edges between CQ^{01}_{n-2} and CQ^{11}_{n-2} of CQ$_n$ into Paths between two components of P$_4$ suffixed by Suf2 and Suf4.

Edges between CQ^{01}_{n-2} and CQ^{11}_{n-2}	Paths between two components of P_4	Dilation
01Pref 010000-11Pref0000	x5x4x3x2x1 Suf2-x3x4x5x2x1 Suf2-x2x5x4x3x1 Suf2-x4x5x2x3x1 Suf2-x1x3x2x5x4 Suf4	4
01Pref 010001-11Pref0011	x3x4x5x2x1 Suf2-x5x4x3x2x1 Suf2-x4x5x3x2x1 Suf2-x1x2x3x5x4 Suf4-x3x2x1x5x4 Suf4	4
01Pref 010010-11Pref0010	x4x5x3x2x1 Suf2-x1x2x3x5x4 Suf4-x2x1x3x5x4 Suf4-x3x1x2x5x4 Suf4	3
01Pref 010011-11Pref0001	x4x3x5x2x1 Suf2-x3x4x5x2x1 Suf2-x2x5x4x3x1 Suf2-x4x5x2x3x1 Suf2-x1x3x2x5x4 Suf4-x2x3x1x5x4 Suf4	5
01Pref 010100-11Pref0100	x2x3x4x5x1 Suf2-x4x3x2x5x1 Suf2-x1x5x2x3x4 Suf4-x3x2x5x1x4 Suf4-x5x2x3x1x4 Suf4	4
01Pref 010101-11Pref0111	x3x2x4x5x1 Suf2-x4x2x3x5x1 Suf2-x1x5x3x2x4 Suf4-x2x3x5x1x4 Suf4-x5x3x2x1x4 Suf4-x3x5x2x1x4 Suf4	5
01Pref 010110-11Pref0110	x4x3x2x5x1 Suf2-x1x5x2x3x4 Suf4-x3x2x5x1x4Suf4	2

01Pref 010111-11*Pref*0101	x4x2x3x5x1*Suf2*-x5x3x2x4x1 *Suf2*-x3x5x2x4x1 *Suf2*-x4x2x5x3x1 *Suf2*-x1x3x5x2x4 *Suf4*-x2x5x3x1x4*Suf4*	5
01Pref 011000-11*Pref*1000	x3x2x5x4x1 *Suf2*-x5x2x3x4x1 *Suf2*-x4x3x2x5x1 *Suf2*- x1x5x2x3x4 *Suf4*-x2x5x1x3x4 *Suf4*	4
01Pref 011001-11*Pref*1011	x5x2x3x4x1 *Suf2*-x4x3x2x5x1 *Suf2*-x1x5x2x3x4 *Suf4*-x5x x1x2x3x4 *Suf4*	3
01Pref 011011-11*Pref*1001	x2x5x3x4x1 *Suf2*-x5x2x3x4x1 *Suf2*-x4x3x2x5 x1 *Suf4*-x1x5x2x3x4 *Suf4*	3
*01Pref*011010-11*Pref*1010	x2x3x5x4x1 *Suf2*-x3x2x x5x4x1 *Suf2*-x4x5x2x3x1 *Suf2*-x1x3x2x5x4 *Suf4*-x3x1x2x5x4 *Suf4*-x5x2x1x3x4 *Suf4*	5
01Pref 011100-11*Pref*1100	x4x5x2x3x1 *Suf2*-x1x3x2x5x4 *Suf4*-x2x3x1x5x4 *Suf4*-x5x1x3x2x4 *Suf4*-x3x1x5x2x4 *Suf4*	4
01Pref 011101- *11Pref*1111	x5x4x2x3x1 *Suf2*-x4x5x2x3x1 *Suf2*-x1x3x2x5x4 *Suf4*-x3x1x2x5x4 *Suf4*-x2x1x3x5x4 *Suf4*	4
01Pref 011110-11*Pref*1110	x2x5x4x3x1*Suf2*-x4x5x2x3x1 *Suf2*-x1x3x2x5x4 *Suf4*-x2x3x1x5x4 *Suf4*-x5x1x3x2x4 *Suf4*	4
*01Pref*1111-11*Pref*1101	x3x5x4x2x1 *Suf2*-x2x4x5x3x1 *Suf2*-x4x2x5x3x1*Suf2*-x1x3x5x2x4 *Suf4*	3

Table 11: One by one embedding edges between CQ^{00}_{n-2} ***and*** CQ^{01}_{n-2} ***of*** **CQ_n** ***into Paths between two components of*** **P_4** ***suffixed by*** **Suf1** ***and*** **Suf2.**

Edges between CQ^{00}_{n-2} and CQ^{10}_{n-2}	Paths between two components of P_4	Dilation
00*Pref* 0000- 01*Pref* 0000	x1x2x3x4x5 *Suf* 1-inv(*Suf*1)x5x4x3x2x *Suf2*	1
00*Pref* 0001- 01 *Pref* 0011	x3x2x1x4x5*Suf* 1-x1x2x3x4x5*Suf* 1- inv(*Suf*1)x5x4x3x2x1*Suf*2-x3x4x5x2x1*Suf*2-x4x3x5x2x1*Suf2*	4
00*Pref* 0010- 01*Pref* 0010	x2x1x3x4x5 *Suf* 2-x1x2x3x4x5*Suf*2-x5x4x3x2x1 *Suf*2-x4x5x3x2x1*Suf2*	3
00*Pref* 0011- 01*Pref* 0001	x2x3x1x4x5 *Suf* 1-x3x2x1x4x5 *Suf* 1-x1x2x3x4x5 *Suf* 1- inv(*Suf*1)x5x4x3x2x1 *Suf* 2-inv1(inv(*Suf*1))x3x4x5x2xx1 *Suf2*	4
00*Pref* 0100- 01*Pref* 0100	x4x3x2x1x5 *Suf* 1-x1x2x3x4x5 *Suf*1-inv (*Suf*1)x5x4x3x2x1 *Suf* 2- inv4(inv (*Suf*1))x2x3x4x5x1*Suf2*	3
00*Pref* 0101- 01 *Pref* 0111	x3x4x2x1x5 *Suf* 1-x1x2x4x3x5 *Suf* 1-x5x3x4x2x1 *Suf* 2-x2x4x3x5x1 *Suf2*-x4x2x3x5x1*Suf2*	4
00*Pref* 0110- 01*Pref* 0110	x2x3x4x1x5 *Suf* 1-x1x4x3x2x5*Suf* 1-x5x2x3x4x1*Suf* 2-x4x3x2x5x1*Suf2*	3
00*Pref* 0111-01*Pref* 0101	x2x3x4x1x5 *Suf* 1-x1x4x3x2x5*Suf* 1-x5x2x3x4x1 *Suf* 2-x4x3x2x5x1 *Suf2*	3
00*Pref* 1000- 01*Pref* 1000	x3x4x1x2x5*Suf* 1-x1x4x3x2x5*Suf* 1-x5x2x3x4x1 *Suf* 2-x3x2x5x4x1 *Suf2*	3
00*Pref* 1001- 01*Pref* 1011	x1x4x3x2x5 *Suf* 1-x5x2x3x4x1 *Suf* 2-x2x5x3x4x1 *Suf2*	2
00*Pref* 1010- 01 *Pref* 1010	x4x3x1x2x5 *Suf* 1-x3x4x1x2x5 *Suf* 1-x1x4x3x2x5 *Suf1*-x5x2x3x4x1 *Suf* 2-x3x2x5x4x1 *Suf2*-x2x3x5x4x1 *Suf2*	5
00*Pref* 1011- 01*Pref* 1001	x4x1x3x2x5 *Suf* 1- *x*1x4x3x2x5 *Suf* 1-x5x2x3x4x1 *Suf2*	2
00*Pref* 1100- 01*Pref* 1100	x2x1x4x3x5 *Suf* 1-x3x4x1x2x5 *Suf* 1-x1x4x3x2x5 *Suf* 1-x5x2x3x4x1 *Suf* 2-x3x2x5x4x1 *Suf* 2-x4x5x2x3x1 *Suf2*	5
00*Pref* 1101- 01*Pref* 1111	x1x2x4x3x5 *Suf* 1-x5x3x4x2x1 *Suf* 2-x3x5x4x2x1 *Suf2*	2
Pref 001110- 01*Pref* 1110	x4x1x2x3x5 *Suf* 1-x3x21x4x5 *Suf* 1-x1x2x3x4x5 *Suf* 1-x5x4x3x2x1 *Suf* 2-x3x4x5x2x1 *Suf2*-x2x5x4x3x1 *Suf2*	5
Pref 001111- *Pref* 011101	x3x1x2x4x5 *Suf* 1-x1*x*3x2x4x5 *Suf* 1-x5x4x2x3x1 *Suf 2*	2

4 DILATIONS OF ONE BY ONE EMBEDDING *N*-DIMENSIONAL CROSSED HYPERCUBE INTO *N*-PANCAKE.

Lemma

The *n*-dimensional crossed hypercube CQ'_n= (*A* , *U'*) has one by one dilation 4 embedding into the *n*-pancake P'_n = (*G',E'*) for any *n*>4.

Proof

We prove this lemma by induction.

Base

For *n*= 4, the Table1, presents all paths between the embedded nodes into P'_4 with dilation 4.

For *n*= 5, The Table2, Table3, Table4, Table5, presents the one by one embedding edges between two P'_3 of any component of P'_n with dilation 4.

Induction hypothesis

Suppose that for $k \leq n$-1, the one to one dilation 4 embedding of CQ'_k into P'_n is true. Let us now prove that is true for k=n.

We have the following cases:

k is odd.

CQ'_k = (A , U') is constructed by two copies of CQ'_{k-1}, the first is prefixed by $0(CQ'_{k-1})$, the second one by $1(CQ'_{k-1})$, such that $X \in A$ *is* denoted by $0Prefa_{k-3}a_{k-2}a_{k-1}$, $0Prefa_{k-3}a_{k-2}a_{k-1}$, $1Pref0a_{k-3}a_{k-2}a_{k-1}$, $1Pref1a_{k-3}a_{k-2}a_{k-1}$ or $Pref_1 1ak_{-3}a_{k-2}a_{k-1}$ or $Pref_1 0ak_{-3}a_{k-2}a_{k-1}$ or or $Pref_2 0a_{k-3}a_{k-2}a_{k-1}$ or $1Pref_2a_{k-3}a_{k-2}a_{k-1}$.The first node of the super node $0(CQ'_{k-1})$ is XX=$x_1x_2x_3x_4$Suf($P_n[n,n]$), we adding only 0 to the prefix of CQ'_{k-1}, that is to say, all edges in $0(CQ'_{k-1})$ are embedded into P_{k-1}[k-1,k-1] (hypothesis of induction). However, we use the different actions depicted by Table1, Table2, Table3, Table4, Table5 In other words the dilation is 4. For the edges between two nodes labeled by $Pref_2 0a_{k-3}a_{k-2}a_{k-1}$ or $Pref_2 1a_{k-3}a_{k-2}a_{k-1}$, we added only 1 to the prefix of CQ'_{k-1}. The first node of the super node $1(CQ'_{k-1})$ is embedded into YY=INV($x_1x_2x_3x_4$Suf($P_{k-1}[k$-1,1]), we apply the same actions of embedding all edges of CQ'_{k-1} into the path in component of P_{k-1} (hypothesis of induction) and we use the different actions cited in Table1, Table2, Table3, Table4, Table5. The dilation in this case is equal 4.

k is even.

CQ'_k= (A, U') is constructed by two copies of CQ'_{k-1}, the first is prefixed by $0(CQ'_{k-1})$, the second one by $1((CQ'_{k-1})$, such that $X \in A$ *is* denoted by $00Prefa_{k-4}a_{k-3}a_{k-2}a_{k-1}$ $01Prefa_{k-4}a_{k-3}a_{k-2}a_{k-1}$,$10Prefa_{k-4}a_{k-3}a_{k-2}a_{k-1}$, $11Pref\ a_{k-4}a_{k-3}a_{k-2}a_{k-1}$. For the edges between nodes labeled $00Prefa_{k-4}a_{k-3}a_{k-2}a_{k-1}$ (first super node of CQ'_k) are embedded into the paths of the first component of $P_{k-1}[k$-1,k-1] named XX= Prem($P_{k-1}[k$-1,k-1]. (Hypothesis of induction).However the dilation is 4.The second one concern edges between nodes of CQ'_k labeled $01Prefa_{k-4}a_{k-3}a_{k-2}a_{k-1}$. This situation is similar that the one by one embedding edges between nodes labeled by $Pref_2 0a_{k-3}a_{k-2}a_{k-1}$ or $Pref_2 1a_{k-3}a_{k-2}a_{k-1}$, we add only 1 to the prefix of $CQ'_{k-1}$and the first node of the super node $1(CQ'_{k-1})$ is embedded into YY=INV(XX) (first node of $P_{k-1}[k$-1,1]).(Hypothesis of induction). However, the dilation is 4. The third situation is the embedding the edges between the nodes labelled $11Prefa_{k-4}a_{k-3}a_{k-2}a_{k-1}$ into the third component of $P_{k-1}[k$-1,2] . We use the same actions except that the first node of this component of CQ'_{k-1} is embedded into ZZ= INV1 (Flip (YY)). (The first of $P_{k-1}[k$-1,2]). (Hypothesis of induction). The dilation is 4.the latest case is idem except that the first node of the forth component of $P_{k-1}[k$-1, 3] TT= INV1 (ZZ).(Hypothesis of induction). However, the dilation is 4.

Theorem

The n-dimensional crossed hypercube CQ_n= (A , U) has one by one dilation 5 embedding into P_n=(G,E) for any n ≥ 5.

Proof

We prove this theorem by induction.

Base

Case, n is odd: n = 5. Table 6, Table 7 presents the different actions of one by one embedding all edges of CQ_n into paths of P_n.

Case when n is even: n = 6. Table 8, Table 9, Table 10, Table 11 presents the cases of different actions of one by one embedding edges of CQ_n into paths of P_n.

Hypothesis of induction

Assume that this theorem holds for $k < n$-1.That is CQ_{k-1} one by one embedding dilation 5 into P_{k-1}.

Now we prove that this is true for $k = n$.

Case 1, k is odd. There are two sub-cases:

One by one embedding edges of $0(CQ'_{k-1})$ and $1(CQ'_{k-1})$ in respectively the first component of $P_k[k$-1,k] and the second $P_k[k$-1,1].In this situation we use the hypothesis of induction except the prefix and suffixes respectively augmented by 0 , k. In the second, we use the same actions like the first situation except Prem(.$P_k[k$-1,1]) = INV(Prem($P_k[k$-1,k])) and the prefix and suffixes respectively augmented by 1, k .(hypothesis of induction). However, the dilation is 5.

One by one embedding edges between $0(CQ'_{k-1})$ and $1(CQ'_{k-1})$, or paths between $P_k[k$-1,k] and $P_k[k$-1,1]. We use the different actions outlined in Table 6, Table7. In all cases, the dilation is 5.

Case 2, k is even. There are two sub-cases:

There are four situations in this sub-case of one by one embedding edges between the same components of CQ_k. The first is between nodes labeled by $00Prefa_{k-4}a_{k-3}a_{k-2}a_{k-1}$, the second one is between nodes labeled $01Prefa_{k-4}a_{k-3}a_{k-2}a_{k-1}$, the third is between nodes labeled $10Prefa_{k-4}a_{k-3}a_{k-2}a_{k-1}$ and the latest is between nodes labeled $11Prefa_{k-4}a_{k-3}a_{k-2}a_{k-1}$.in the all situation, we use $00Prefa_{k-4}a_{k-3}a_{k-2}a_{k-1}$ In this situation we use the hypothesis of induction except the prefix and suffixes respectively augmented by 0 , k. In the second, we use the same actions like the first situation except Prem(.$P_k[k$-1,1]) = INV(Prem($P_k[k$-1,k])) and the prefix and suffixes respectively augmented by 1 , k .(Hypothesis of induction). However, the dilation is 5. Idem for the remaining situation, except the first node of the third used component is Prem(.$P_k[k$-1,2) = INV(INV1(Prem($P_k[k$-1,1]))) and the first node of the forth used component is Prem($P_k[k$-1,3])= INV(Prem($P_k[k$-1,2])).

The second sub-case is one by one embedding edges between $00(CQ'_{k-2})$ and $01(CQ'_{k-2})$ $00(CQ'_{k-2})$ and $10(CQ'_{k-2})$, $10(CQ'_{k-2})$ and $11(CQ'_{k-2})$, $01(CQ'_{k-2})$ and $11(CQ'_{k-2})$ onto respectively paths between $P_k[k$-1,k] and $P_k[k$-1,1], $P_k[k$-1,2] and $P_k[k$-1,3], $P_k[k$-1,2] and $P_k[k$-1,4].and finally between $P_k[k$-1,1] and $P_k[k$-1,4]. We use respectively the different actions outlined in Table 11, Table 8, Table 9 Table10, .In all cases the dilation is 5.

5 CONCLUSION

It is both practically significant and theoretically interesting to investigate the embeddability of different architecture into pancake (ref).in this paper, the main purpose is the one by one dilation 5 embedding crossed hypercube into Pancake. The study of dilation of this function is explained in two steps. The first step is the one by one dilation 4 embedding of all edges in the same P_4 of any super node of P_n as proved by lemma. The second step is the general one by one dilation 5 embedding of all edges of crossed hypercube into paths between two super nodes of Pancake is proved by theorem.

In the future of this work, it is more interesting to study the fault-tolerant embedding of crossed hypercube into Pancake.

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